TRAFFIC SIGNAL SETTINGS

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FOREWORD

IT HAS BEEN estimated that delays to traffic at signal-controlled intersections in Great Britain might amount to 100 million vehicle-hours a year. Whilst much of this delay is inevitable with single-level intersections it is clearly important to reduce it to a minimum: this means that the signals should be correctly set. This paper presents the results of research carried out by the Road Research Laboratory into traffic-signal settings and expected delay to vehicles. It is the first part of a larger investigation and deals with the problem of fixed-time signals. Although 6 out of 7 signals in Great Britain are of the vehicle-actuated type they behave effectively as fixed-time lights when traffic is heavy, i.e. when delays are greatest. The results thus have application to those signals and also to linked systems of lights which work on fixed cycles.

The investigation has been made using a high-speed electronic computer to simulate the behaviour of traffic at the signals. A formula for delay has been deduced and this has been used to determine two simple relations for the green times and cycle time that give the least delay to all vehicles using the intersection.

As well as helping in the setting of signals, the results can be used to assess the gains to be expected at signal-controlled intersections from road improvements, from banning right-turners, and from banning parked vehicles.

W. H. GLANVILLE,
Director of Road Research

ROAD RESEARCH LABORATORY
October, 1957

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Traffic Signal Settings

INTRODUCTION

EXTENSIVE use is made of traffic signals for the control of conflicting streams of both vehicular and pedestrian traffic. In Great Britain alone there are more than 4000 signal installations, and about 1000 of these are in the Greater London Area. Moreover, the usage of traffic signals is increasing; for instance, 23 new sets of signals were installed in London in 1955.

Signals are of two main types, fixed-time and vehicle-actuated. With fixed-time signals the sequence of lights shown on each approach to the intersection has a cycle of fixed duration, and each signal indication appears for a fixed period. With vehicle-actuated signals, however, approaching vehicles are detected by devices in the road and the duration of the green period on each approach varies with the traffic demand. Even though vehicle-actuated signals are replacing the fixed-time variety in Great Britain, fixed-time signals number about one in seven, and in some countries, e.g. the U.S.A., fixed-time signals are much more numerous than vehicle-actuated ones. A more detailed description of signals can be found in "Automatic street traffic signalling" by Harrison and Priest[1].

Since the introduction in 1926 of automatic signals much work has been done, particularly in Great Britain and the U.S.A., on methods for setting traffic signals (see Clayton[2], Matson[3]). Almost all of this work was devoted to fixed-time signals, and formulae for cycle time and the ratio of the green times were deduced. With regard to delay, however, none of the formulae was really applicable to practical conditions, being based on the assumption that traffic arrives at an intersection at a uniform rate[4]. It has been shown by Adams [5] that, generally, the arrival times of vehicles are distributed at random, and this has been substantiated by the results of experiments carried out by the Road Research Laboratory. No satisfactory method was available for calculating the delay when the flow was assumed to be random, and purely theoretical work in this field has been of a very limited character. It was thus very difficult to make a realistic assessment of any road improvements at signalled junctions or of changes brought about by prohibiting waiting vehicles, right-turning vehicles, etc.

This paper presents results of research conducted by the Road Research Laboratory into the delays to vehicles at fixed-time traffic signals and into the optimum settings of such signals. The methods developed in this study can be applied both to fixed-time working and to vehicle-actuation, but, whereas at fixed-time signals each approach can reasonably be treated separately, an intersection controlled by vehicle-actuated signals must be dealt with as a whole.

The results obtained provide a more definite basis for setting fixed-time signals than any which have so far been published. Perhaps of more practical importance, however, is their application to those vehicle-actuated signals where the green periods owing to heavy traffic demands are frequently running to maximum, giving in effect fixed-time control. Such signals are very numerous in large cities where the traffic flow is usually heavy between about 8 a.m. and 7 p.m. Furthermore, linked systems of traffic signals often work on a fixed cycle which has been set to meet the requirements of the main intersection of the system and the desired speed of the traffic along the road.

A glossary of terms and symbols used in this paper is given in Appendix 1. In general, when formulae are quoted, no units are mentioned since the relations hold between the physical quantities concerned. When interpreting a formula it is, of course, necessary to use the same units of time throughout.

All important equations, and any other to which reference is made, are numbered in order. The numbering of equations in each of the Appendices starts at one, and is preceded by the number of the appendix, e.g. the fifth equation of Appendix 3 is numbered (3·5).

METHOD

The first objective in the research was to determine delays. Since a theoretical calculation of delay is very complex and direct observation of delay on the road is complicated by uncontrollable variations, it was decided to use a method whereby the events on the road are reproduced in the laboratory by means of some machine which simulates the behaviour of traffic and traffic signals at an intersection. This technique enables the variables to be controlled, e.g. the same traffic can be used for several settings of the signals. The simulation was carried out using the Pilot Model ACE (Automatic Computing Engine) at the National Physical Laboratory through the kind co-operation of the Director. The high attainable speed of the simulation process over 'life' speed is an additional advantage. This method is described in more detail in Appendix 2.

To reproduce in the Laboratory the events on the road some record of actual or artificially produced traffic is required. Traffic may be considered as arriving at random[5] provided that the point at which it is observed is some distance from a disturbing factor such as a controlled intersection. This was assumed in

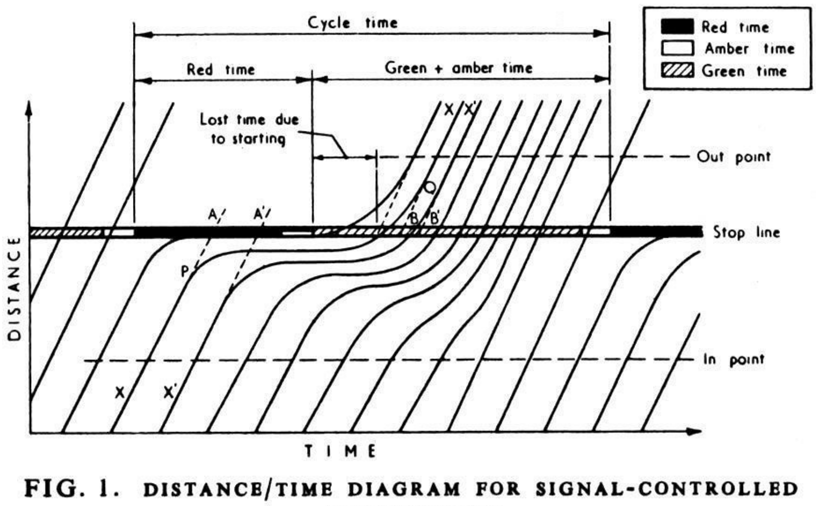

FIG. 1. DISTANCE/TIME DIAGRAM FOR SIGNAL-CONTROLLED INTERSECTION

producing artificial traffic records for this experiment. The delay to the traffic is computed by counting the number of vehicles in the queue at fixed intervals of time and multiplying this number by the value of the time interval. For example, if there are n vehicles in the queue and the queue is counted every u seconds then in a particular interval of time the total delay amounts to nu vehicle-seconds. The total delay over a long period is obtained by summing this product at every interval. The number of arrivals is also recorded and the average delay is obtained by dividing the total delay by the number of arrivals.

When the green period commences a certain time elapses while vehicles are accelerating to normal running speed (see Fig. 1), but after a few seconds the queue discharges at a more or less constant rate, called the saturation flow. If there is still a queue at the end of the green period some vehicles will make use of the amber period to cross the intersection. In these circumstances traffic moves on both green and amber signals but the discharge rate is less than the saturation flow both at the beginning and at the end of the right-of-way period, as shown in Fig. 2. The green and amber periods together $(k+a)$ may be re-

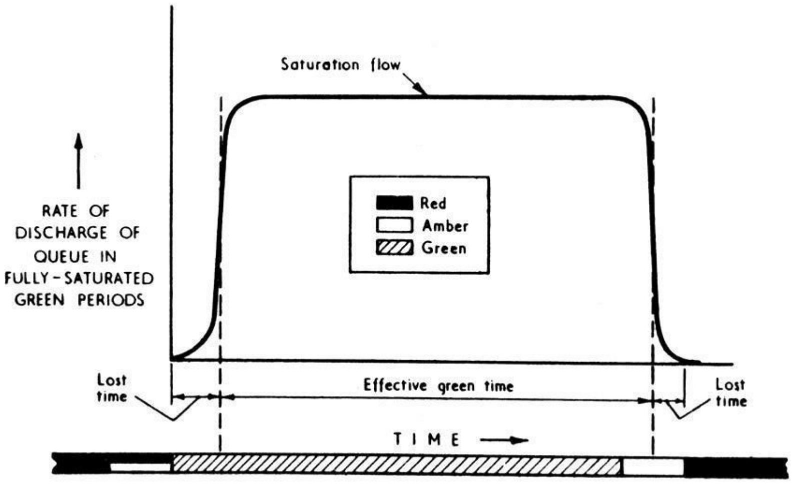

FIG 2. VARIATION OF DISCHARGE RATE OF QUEUE WITH TIME IN A FULLY-SATURATED GREEN PERIOD

placed by an 'effective' green (g) and a 'lost' time (l), such that the product of the effective green and the saturation flow is equal to the correct number of vehicles (say, b) discharged from the queue on the average in a saturated green period (i.e. a green period during which the queue never clears).

Thus, $k+a=g+l$

and $b=gs$

where s is the saturation flow.

It is assumed in the computation that the saturation flow is constant; in practice it may vary within a cycle and between cycles, but it has been found that the results obtained here, assuming a constant saturation flow, agree well with values observed over a fairly large number of cycles.

In the simulation experiment the signal sequence on any approach is thus reduced to an 'effective' green period and a 'red' period which comprises all the times when traffic cannot run, i.e. red and red-with-amber periods plus 'lost' time.

In the investigation described in this paper the delay to traffic using a single approach to an intersection controlled by fixed-time traffic signals was computed over a range of values of green times, cycle times, traffic flow and saturation flow covering most practical possibilities.

AVERAGE DELAY PER VEHICLE

It was found that the results of the computation could be expressed to a close approximation by the equation

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + \frac{x^2}{2q(1-x)} - 0{\cdot}65 \left(\frac{c}{q^2}\right)^{\frac{1}{3}} x^{(2+5\lambda)} \ldots\ldots(1)$$

where d = average delay per vehicle on the particular arm of the intersection

c = cycle time

λ = proportion of the cycle which is effectively green for the phase under consideration (i.e. g/c)

q = flow

s = saturation flow

and x = the degree of saturation. This is the ratio of the actual flow to the maximum flow which can be passed through the intersection from this arm, and is given by $x = q/\lambda s$

If d and c are in seconds, q and s are in vehicles per second.)

The expression for delay was not derived entirely theoretically. Terms 1 and 2 each have a theoretical meaning but the last term is purely empirical. The first term of equation (1) is the expression for the delay when the traffic can be considered to be arriving at a uniform rate[4]. Although the agreement between computed delays and those derived from this term is fairly good at low flows, as shown by Fig. 3, it is not so at higher values where the computed delays, owing to the random nature of the arrivals, are far in excess of values calculated from this term only of the equation. The second term of equation (1) makes some allowance for the random nature of the arrivals. It is an expression for the delay experienced by vehicles arriving randomly in time at a 'bottleneck', queueing up, and leaving at constant intervals[6]. If the delay is represented by these two terms, i.e. if

$$d = \frac{c\,(1-\lambda)^2}{2\,(1-\lambda x)} + \frac{x^2}{2q\,(1-x)} \ldots\ldots(2)$$

a fair agreement with the computed delays is obtained at all levels of flow (see Fig. 3).

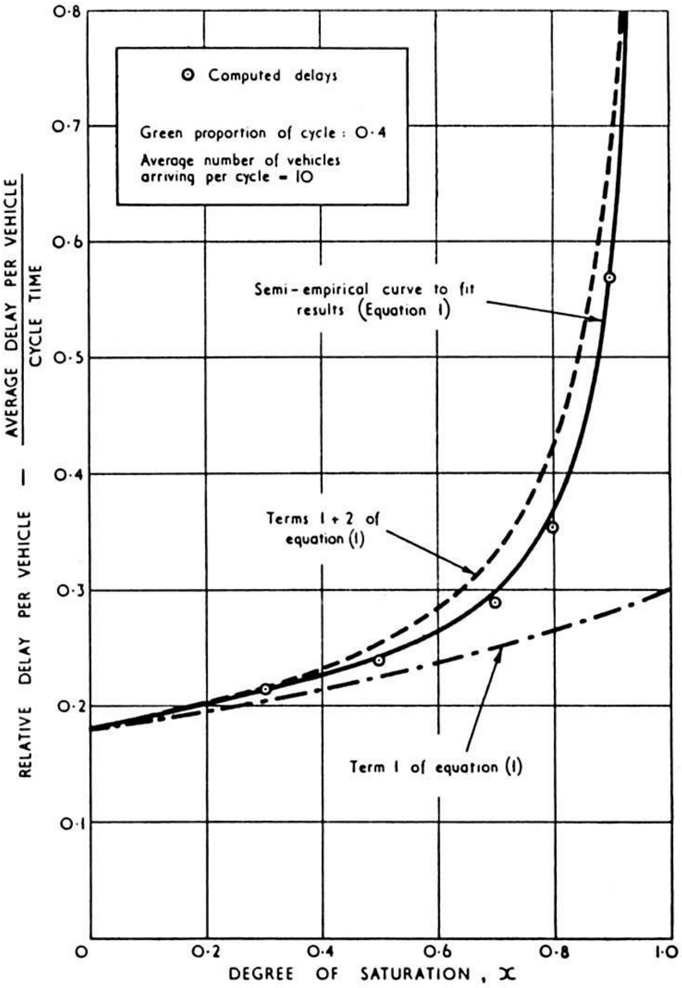

FIG. 3. TYPICAL FIXED-TIME DELAY CURVE

When the empirical correction term (third term of equation (1)) is added to the above expression the agreement is improved and the complete expression gives a closer fit for all values of flow. Since the value of the correction term is generally in the range 5 to 15 per cent of d the delay for most practical purposes can be represented adequately by 9/10 of that given by equation (2).

The delay formula has been tested under actual road conditions at several intersections with fixed-time and vehicle-actuated signals and the variation between observed and calculated values was no greater than would be expected due to random fluctuations (see Fig. 4).

An estimate of the average delay per vehicle may be required by highway engineers, e.g. to evaluate the economic advantages of road improvements. To enable the expected delay to be estimated more easily, equation (1) has been rewritten as

$$d = cA + \frac{B}{q} - C \ldots\ldots (3)$$

where $A = \frac{(1-\lambda)^2}{2(1-\lambda x)}$, $B = \frac{x^2}{2(1-x)}$ and C is the correction term.

A and B have been tabulated (see Tables 1 and 2) and C has been calculated as a percentage of the first two terms in (3) and is given in Table 3 in terms of x, λ and M where $M = qc$.

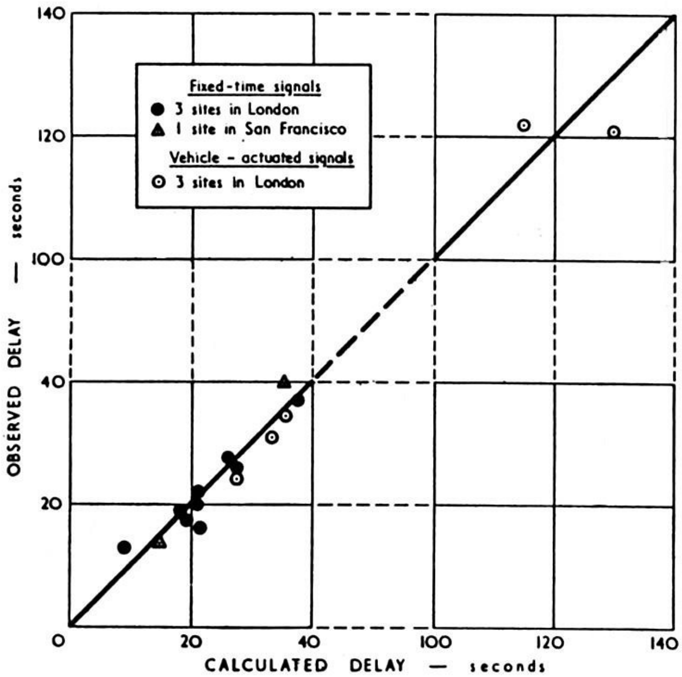

FIG. 4. COMPARISON OF OBSERVED AND CALCULATED DELAYS

In addition, equation (1) as a whole has been tabulated over an extensive range of values. These tables have not been included in this paper, but they can be obtained, if desired, by application to the Director of Road Research.

Information available on lost time suggests a value of about 2 seconds per phase plus any all-red periods. Since lost time depends on gradients, type of traffic, etc. its value will vary from site to site and even at the same site will probably vary with time of day. In fact, values ranging from ½ to 7 seconds have been observed in extreme cases. However, the absolute value of lost time is not too important as far as delay is concerned provided that the product of effective green and saturation flow gives the correct average number of vehicles discharged from the queue in fully saturated green periods. In the absence of more reliable information therefore it is suggested that its value is assumed to be 2 seconds per phase (excluding any all-red periods). More research is being carried out by the Laboratory into the effect of traffic behaviour and road layout on lost time.

TABLE 1

TABULATION OF $A = \dfrac{(1-\lambda)^2}{2(1-\lambda x)}$

x \ λ	0·1	0·2	0·3	0·35	0·40	0·45	0·50	0·55	0·60	0·65	0·70	0·80	0·90
0·1	0·409	0·327	0·253	0·219	0·188	0·158	0·132	0·107	0·085	0·066	0·048	0·022	0·005
0·2	0·413	0·333	0·261	0·227	0·196	0·166	0·139	0·114	0·091	0·070	0·052	0·024	0·006
0·3	0·418	0·340	0·269	0·236	0·205	0·175	0·147	0·121	0·098	0·076	0·057	0·026	0·007
0·4	0·422	0·348	0·278	0·246	0·214	0·184	0·156	0·130	0·105	0·083	0·063	0·029	0·008
0·5	0·426	0·356	0·288	0·256	0·225	0·195	0·167	0·140	0·114	0·091	0·069	0·033	0·009
0·55	0·429	0·360	0·293	0·262	0·231	0·201	0·172	0·145	0·119	0·095	0·073	0·036	0·010
0·60	0·431	0·364	0·299	0·267	0·237	0·207	0·179	0·151	0·125	0·100	0·078	0·038	0·011
0·65	0·433	0·368	0·304	0·273	0·243	0·214	0·185	0·158	0·131	0·106	0·083	0·042	0·012
0·70	0·435	0·372	0·310	0·280	0·250	0·221	0·192	0·165	0·138	0·112	0·088	0·045	0·014
0·75	0·438	0·376	0·316	0·286	0·257	0·228	0·200	0·172	0·145	0·120	0·095	0·050	0·015
0·80	0·440	0·381	0·322	0·293	0·265	0·236	0·208	0·181	0·154	0·128	0·102	0·056	0·018
0·85	0·443	0·386	0·329	0·301	0·273	0·245	0·217	0·190	0·163	0·137	0·111	0·063	0·021
0·90	0·445	0·390	0·336	0·308	0·281	0·254	0·227	0·200	0·174	0·148	0·122	0·071	0·026
0·92	0·446	0·392	0·338	0·312	0·285	0·258	0·231	0·205	0·179	0·152	0·126	0·076	0·029
0·94	0·447	0·394	0·341	0·315	0·288	0·262	0·236	0·210	0·183	0·157	0·132	0·081	0·032
0·96	0·448	0·396	0·344	0·318	0·292	0·266	0·240	0·215	0·189	0·163	0·137	0·086	0·037
0·98	0·449	0·398	0·347	0·322	0·296	0·271	0·245	0·220	0·194	0·169	0·143	0·093	0·042

TABLE 2

TABULATION OF $B = \frac{x^2}{2(1-x)}$

x	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09
0·1	0·006	0·007	0·008	0·010	0·011	0·013	0·015	0·017	0·020	0·022
0·2	0·025	0·028	0·031	0·034	0·038	0·042	0·046	0·050	0·054	0·059
0·3	0·064	0·070	0·075	0·081	0·088	0·094	0·101	0·109	0·116	0·125
0·4	0·133	0·142	0·152	0·162	0·173	0·184	0·196	0·208	0·222	0·235
0·5	0·250	0·265	0·282	0·299	0·317	0·336	0·356	0·378	0·400	0·425
0·6	0·450	0·477	0·506	0·536	0·569	0·604	0·641	0·680	0·723	0·768
0·7	0·817	0·869	0·926	0·987	1·05	1·13	1·20	1·29	1·38	1·49
0·8	1·60	1·73	1·87	2·03	2·21	2·41	2·64	2·91	3·23	3·60
0·9	4·05	4·60	5·28	6·18	7·36	9·03	11·5	15·7	24·0	49·0

Example

The flow on a particular arm of a junction is 600 vehicles per hour and the signal settings are 29 seconds green, 3 seconds amber and 60 seconds cycle time. It is observed that on the average 15·0 vehicles are discharged in a fully saturated green period. If we assume that starting delays, etc. are responsible for 2 seconds of each green-plus-amber period then 15 vehicles are discharged in an effective green time of 30 seconds, i.e. $s=1800$ vehicles per hour.

We have,

$$\lambda = \frac{g}{c} = \frac{30}{60} = 0{\cdot}5$$

$$x = \frac{q}{\lambda s} = \frac{600}{0{\cdot}5(1800)} = 0{\cdot}667$$

and $M = 10$

From Table 1, $A = 0{\cdot}187$
From Table 2, $B = 0{\cdot}667$
From Table 3, $C =$ 9 per cent of the first two terms
Using equation (3)

$$d = 60\,(0{\cdot}187) + \frac{0{\cdot}667}{600/3600} - C$$

$$= 11{\cdot}2 + 4{\cdot}0 - C$$
$$= 15{\cdot}2 - 1{\cdot}4$$
$$d = 13{\cdot}8 \text{ seconds}$$

TABLE 3

CORRECTION TERM OF EQUATION (1) AS A PERCENTAGE OF THE FIRST TWO TERMS

x	λ \ M	2·5	5	10	20	40
0·3	0·2	2	2	1	1	0
	0·4	2	1	1	0	0
	0·6	0	0	0	0	0
	0·8	0	0	0	0	0
0·4	0·2	6	4	3	2	1
	0·4	3	2	2	1	1
	0·6	2	2	1	1	0
	0·8	2	1	1	1	1
0·5	0·2	10	7	5	3	2
	0·4	6	5	4	2	1
	0·6	6	4	3	2	2
	0·8	3	4	3	3	2
0·6	0·2	14	11	8	5	3
	0·4	11	9	7	4	3
	0·6	9	8	6	5	3
	0·8	7	8	8	7	5
0·7	0·2	18	14	11	7	5
	0·4	15	13	10	7	5
	0·6	13	12	10	8	6
	0·8	11	12	13	12	10
0·8	0·2	18	17	13	10	7
	0·4	16	15	13	10	8
	0·6	15	15	14	12	9
	0·8	14	15	17	17	15
0·9	0·2	13	14	13	11	8
	0·4	12	13	13	11	9
	0·6	12	13	14	14	12
	0·8	13	13	16	17	17
0·95	0·2	8	9	9	9	8
	0·4	7	9	9	10	9
	0·6	7	9	10	11	10
	0·8	7	9	10	12	13
0·975	0·2	8	9	10	9	8
	0·4	8	9	10	10	9
	0·6	8	9	11	12	11
	0·8	8	10	12	13	14

*M is the average flow per cycle $= qc$

OPTIMUM SETTINGS OF FIXED-TIME SIGNALS

In general, all approaches belonging to the same phase will have the same green period even though the traffic requirements of the approaches may be different. It will be shown later that each phase can be represented by one approach only —the one with the highest ratio of flow to saturation flow. Let this ratio be denoted by the symbol y.

In deriving the optimum green times and cycle time the empirical correction term of the delay equation was neglected since some preliminary calculations showed that its variation with respect to these quantities was slight.

Green times

It is suggested in the traffic engineering handbook[7] that the least delay to traffic is obtained when the green periods of the phases are in proportion to the corresponding ratios of flow to saturation flow, assuming this ratio to be the same for all arms of the same phase. This division of the cycle time makes the capacity of the phases* proportional to the average flows of the phases. That this is approximately the best division of the cycle time is shown by the examples given below.

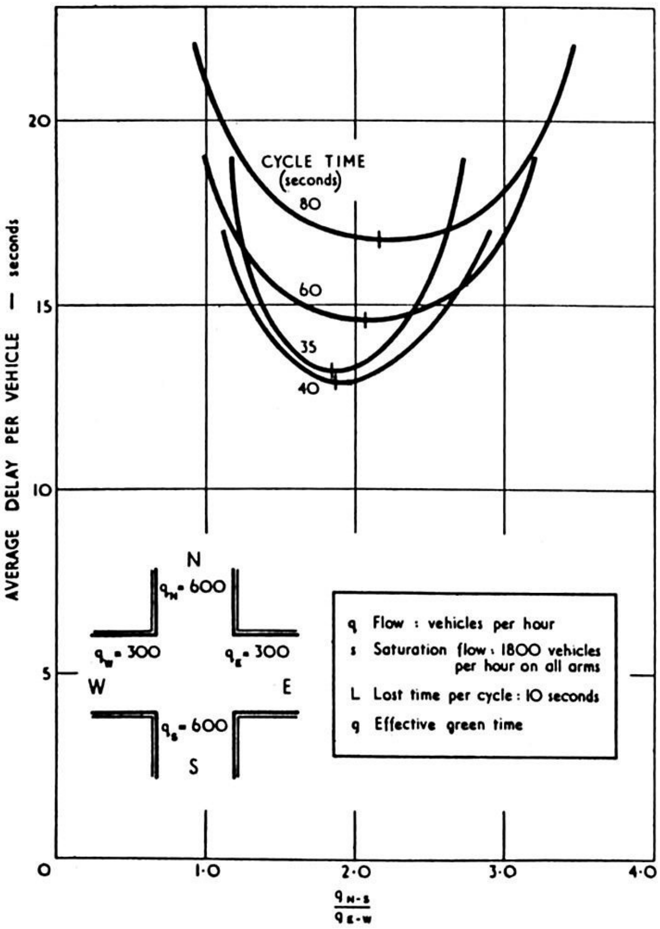

FIG. 5. EFFECT ON DELAY OF VARIATION OF RATIO OF GREEN PERIODS

* Capacity is the maximum amount of traffic that can be discharged in the green period.

The total delay experienced by vehicles at an intersection was calculated using the approximate form of the delay equation (equation (2)). The lost time was assumed to be 10 seconds per cycle and delays were calculated for a variety of cycle times and ratios of the effective green times. A typical result is shown in Fig. 5, where the ratio of the y values is 2·0. It can be seen that the best ratio of the effective green times is between 1·88 and 2·17 over a range of cycle times of 35 to 80 seconds. Many other examples have been worked out in a similar manner and the results in a few cases, including the example illustrated in Fig.5, are given in Table 4. In each case the ratio of the green times which gave the least delay was obtained from curves such as those plotted in Fig. 5, and in cases 1 to 4, from the curves corresponding to the most favourable cycle times. The ratio of the y values can be seen to be within a few per cent of the best ratio of the effective green times and the extra delay (over the theoretical minimum) that results from making the effective green ratio equal to the y ratio is very small, as shown in the last column of Table 4.

The problem has also been treated mathematically and it has been shown that, for a two-phase intersection, where y_1/y_2 is greater than unity the effective green ratio should, strictly, be slightly less than the y ratio. However, the difference is negligible in most cases. Futhermore, it has been shown that, where the two arms of a single phase have different values of the ratio q/s (so far they have been assumed to be equal), approximately minimum overall delay is still

TABLE 4

OPTIMUM GREEN PROPORTIONS FOR VARIOUS TRAFFIC PATTERNS
(Lost time of 10 seconds per cycle)

Case No.	Flow (vehicles/hour) / Saturation flow: N-S phase		E-W phase		Cycle length (seconds)	$\frac{g_{N-S}}{g_{E-W}}$ for minimum delay (computed results)	$\frac{y_{N-S}}{y_{E-W}}$	Percentage increase in delay over theoretical minimum values, when effective green times are set in proportion to y values
	N	S	E	W				
1	900/1800	900/1800	300/1800	300/1800	60	2·80	3·00	2
2	600/1200	600/1200	600/2400	600/2400	80	1·89	2·00	3
3	600/1800	600/1800	200/600	200/600	60	1·03	1·00	1
4	750/1800	200/1800	400/1800	400/1200	80	1·235	1·25	0
5	600/1800	600/1800	300/1800	300/1800	35	1·88	2·00	1
					40	1·90	2·00	1
					60	2·07	2·00	0
					80	2·17	2·00	1

obtained by dividing the cycle according to the y values even though the q/s ratio of other arms may have any values between O and y.

Summing up therefore, it may be stated that for all practical purposes the simple rule of setting the effective green times in proportion to the y values of the phases is adequate, particularly since some of the variables, e.g. saturation flow and lost time, can usually only be estimated approximately. After calculating the effective green times according to the simple rule the lost time per phase should be added and the amber period substracted from these values giving the controller settings of the green times. The phase having the lower green time may be given preference when rounding off the controller settings to whole numbers of seconds.

Cycle time

In deriving an expression for the optimum cycle time it was assumed that the effective green times of the phases were in the ratio of their respective y values.

Equation (2) representing the delay to traffic on one arm of an intersection was modified to cover the general case of an intersection with n phases, and the modified equation differentiated with respect to cycle time, to determine the

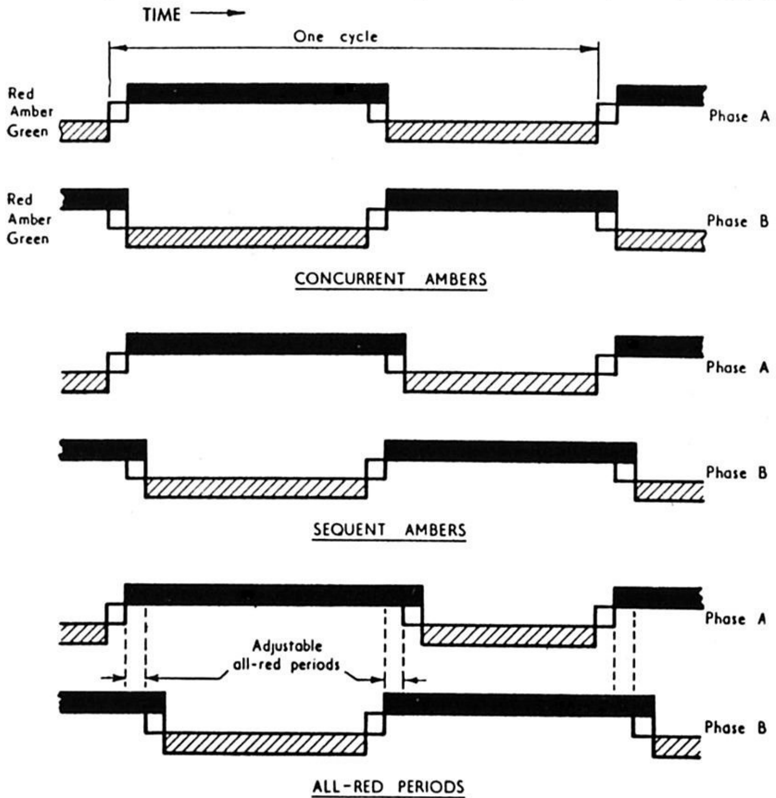

FIG. 6. DIAGRAM SHOWING POSSIBLE ASPECTS OF THE SIGNALS AT A 2-PHASE INTERSECTION

value of cycle time which gives the least delay to all traffic using the intersection. The derivation of the optimum cycle-time formula (equation (3·12)) is given in Appendix 3. It was considered that this formula was too complicated for most purposes and a simple approximation was therefore derived (see Appendix 3). It is given by

$$c_0 = \frac{1{\cdot}5L + 5}{1 - Y} \quad \text{seconds} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where Y is the sum of the y values and refers to the intersection as a whole and L is the total lost time per cycle in seconds. The lost time can be expressed by

$$L = nl + R$$

where n is the number of phases

l is the average lost time per phase (excluding any all-red periods or sequent ambers)

R is the time during each cycle when all signals display red (including red-with-amber) simultaneously (see Fig. 6).

Equation (4) should be adequate for most practical situations. However, if it is desired to use the more exact formula for the determination of the cycle length, this may be done fairly simply by carrying out the procedure given in Appendix 3, where the equations have been broken down into a series of simple operations; some of these remain in their arithmetical form whilst the others have been tabulated and converted into graphical form.

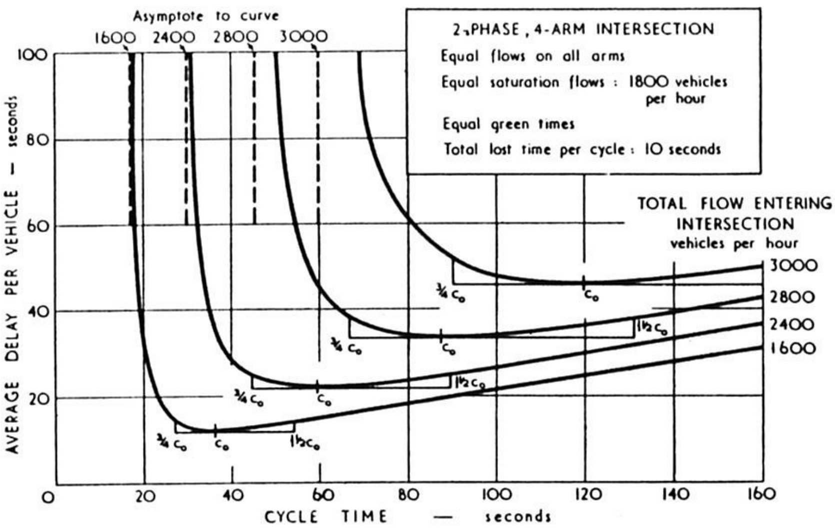

FIG. 7. EFFECT ON DELAY OF VARIATION OF THE CYCLE LENGTH

Several examples of hypothetical intersections (including some fairly extreme cases) have been studied to show the effect of changes in cycle time on the delay. For a symmetrical intersection values of delay have been deduced from equation (1) and are shown in Fig. 7 plotted against the cycle length for several values of the total flow entering the intersection. The cycle time giving the least delay is in each case approximately twice the least possible cycle which will just allow the traffic to pass through (although with long delays). The latter is called the minimum cycle, and is, of course, the vertical asymptote to the delay/cycle-time curve. This simple relation between the optimum cycle and the minimum cycle was used in deriving the more accurate expressions for the optimum cycle time in Appendix 3.

When traffic is of a truly random character the minimum cycle is associated with infinite delay. For uniform flow it is the cycle such that all traffic arriving during the cycle can just be discharged during the green. However, even in this case it represents an unstable situation, since any increase in flow, however slight, will result in the formation of a steadily increasing queue. Previous formulae for cycle times have been based on the minimum cycle for uniform arrivals, the constants being adjusted empirically to account for the difference between random and uniform flow[2].

From graphs such as Fig. 7, it was found that in most practical cases the delay for cycle times within the range ¾ to 1½ times the optimum value is never more than 10 to 20 per cent greater than that given by the optimum cycle. This fact can be used in deducing a compromise cycle time when the level of flow varies considerably throughout the day. It would be better either to change the cycle time to take account of this, or, as is more common, to use vehicle-actuated signals. However, for a single setting of fixed-time signals the simple approximate method outlined below may be used.

(i) Calculate the optimum cycle for each hour of the day when the traffic flow is medium or heavy, e.g. between the hours of 8 a.m. and 7 p.m. and average over the day.

(ii) Evaluate three-quarters of the optimum cycle calculated for the heaviest peak hour.

(iii) Select whichever is greater for the cycle time.

It is suggested as a reasonable procedure that the division of the available green time ($c_0 - L$) should be in proportion to the average y values for peak periods only, i.e.

$$\frac{g_1}{g_2} = \frac{(y_1)_{PEAK}}{(y_2)_{PEAK}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where $(y_1)_{PEAK}$ is the average y value during the peak periods for phase 1 and $(y_2)_{PEAK}$ that for phase 2.

Example. Measurements of flow and saturation flow at a particular 2-phase, 4-arm intersection were as follows:

		North	*South*	*East*	*West*
Flow (q)	vehicles per hour ..	600	450	900	750
Saturation flow (s)	,, ,, ,, ..	2400	2000	3000	3000
Ratio (q/s)	.. ..	0·250	0·225	0·300	0·250
y values ..	.. ..	0·250		0·300	
Lost time					
Starting delays..	.. .. ..	2 seconds per phase			
All-red periods	.. .. ..	3 seconds at each change of right of way			

The total time per cycle when red or red-with-amber signals are showing to all phases is 12 seconds (see Fig. 6). The total lost time per cycle is therefore 16 seconds.

Substituting in equation (4) we have

$$c_0 = \frac{1 \cdot 5\,(16) + 5}{1 - 0 \cdot 250 - 0 \cdot 300} = \frac{29}{0 \cdot 450} = 64 \text{ seconds}$$

In one cycle there will be 64 — 16 = 48 seconds total effective green time which should be divided between the north-south, east-west phases in the ratio of 0·250 : 0·300. The effective green times of the phases are therefore 22 seconds N-S and 26 seconds E-W, and the controller settings will be obtained by including the 2 seconds lost time in each phase, viz.

N-S phase: 21 seconds green plus 3 seconds amber,
E-W phase: 25 seconds green plus 3 seconds amber.

Under light traffic conditions the 'optimum' cycle time as deduced from equation (4) may be very short. From a practical point of view including safety considerations, it is undesirable to have cycles less than, say, about 25 seconds. In any case, the assumptions made in deducing the optimum cycle time formula may no longer hold when the green periods are so short.

MISCELLANEOUS RESULTS

Degree of saturation

It is shown in Appendix 4 that for optimum division of the cycle time the degree of saturation should be the same for all phases of the intersection. In this calculation, we have considered only one arm from each phase—the one with the highest q/s value. The degree of saturation for optimum settings of the controller appears to be independent of the amount of lost time per cycle, depending only on Y. It is given by equation (4·7) in Appendix 4 as

$$x_0 = \frac{2Y}{1 + Y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Average delay for the whole intersection

The average delay to all vehicles using an intersection has been deduced for optimum settings of the controller. The steps of the calculation are shown in Appendix 4 where the average delay per vehicle is given by equation (4·12) as

$$\bar{d} = \frac{c_0}{2} \left\{ 1 - \frac{\sum_{1}^{n'} y_r\, q_r}{YQ} + \frac{2n' Y^2}{LQ(1 + Y)} \right\} \quad \text{approximately} \ldots.(7)$$

where n' is the number of approaches to the intersection. This expression applies only to junctions where all arms of any one phase have approximately the same ratio of flow to saturation flow. The expression does not include the empirical correction term of equation (1), but this can be taken into account, approximately, by reducing $\bar{d}$ by about 10 per cent.

It may be simpler to use equation (7) to estimate the average overall delay for an intersection working under optimum conditions than to calculate the delay for each phase individually and average over the number of vehicles. The value deduced from the equation may be compared with existing delays at a particular intersection to indicate the magnitude of possible improvements which may be obtained from retiming the signals.

For a constant ratio of flows (phase 1 to phase 2) the average delay under optimum conditions varies in direct proportion to the optimum cycle. Thus, if a programme alters the cycle time to maintain optimum conditions at an intersection as the flows vary (either according to a pre-arranged plan or by using a detector to count the traffic and effect the necessary changes in controller settings), the changes in cycle time should be accompanied by proportionate changes in average delay, provided the ratio of flows remains fairly constant.

A few examples of the use of equation (7) are shown in Appendix 4.

Number of fully saturated green periods

During the computation of delay, the queue at the beginning of each green period was recorded and a frequency distribution of queues was obtained for each set of variables. The number of cycles during which the queue never cleared was deduced theoretically from a knowledge of the frequency distribution of the queue at the beginning of the green period and the probability that a given queue would be reinforced by sufficient arrivals during the green period to prevent it from disappearing before the end of the green period. These values are shown in Fig. 8. The number of cycles running to saturation can provide a quick and easy check on the degree of saturation.

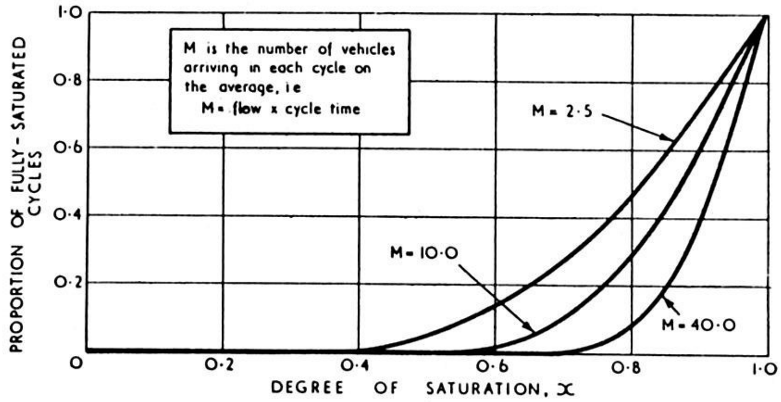

FIG. 8. THE PROPORTION OF CYCLES WHICH RUN TO SATURATION

Experiments were carried out at three intersections in the London area to test this result. The observed degree of saturation over a period of a few hours (obtained by dividing the flow by the product of saturation flow and the proportion of green time) compared satisfactorily with values obtained from Fig. 8, knowing the number of cycles which were fully saturated and the average number of vehicles which arrived in each cycle. The comparison is shown in Table 5.

Queues

In designing signal control at an intersection it is important to know what extensions of the queue are likely to occur, particularly if other intersections are fairly close.

As already stated, a frequency distribution of queues was obtained from the computation. For light flows the average queue at the beginning of the green period was almost always equal to the number of vehicles which had arrived during the red period, and hence followed a Poisson distribution, but for heavier flows the distribution had a considerably longer tail. The point at which an appreciable departure from the Poisson distribution occurs is that at which the majority of the cycles are fully saturated.

The queue at the beginning of the green period, is, in general, the maximum queue in each cycle. It is, to a large extent, a function of the delay at the intersection. Owing to the random nature of the arrivals there is usually a mixture of saturated and unsaturated cycles, and in deriving an expression for the queue length it was found necessary to deal with these types of cycles separately, combining the results to cover the general case (see Appendix 5). The average queue at the beginning of the green period is given approximately by equation (5·8) as:

$$N = \left(\frac{qr}{2} + qd\right) \text{ or } qr \quad \dots\dots (8)$$

whichever is the larger, where r is the red time, q is the flow, and d is the average delay per vehicle.

Several values of queue length, determined from equation (8), are shown in Table 6 to be in good agreement with computed values.

TABLE 5

TEST EXPERIMENTS: FIXED-TIME SIGNALS

Location	Queues (N) (average number of vehicles)		Degree of saturation (%)	
	Calc.	Obs.	From Fig. 8	Obs.
Old St/City Rd				
North arm	8·9	9·6	89	89
East arm	6·0	6·3	88	87
Clerkenwell Rd/St John's St				
West arm	7·0	7·2	88	86
South arm	1·7	2·0	—	—
Putney High St/Upper Richmond Rd				
West arm	5·4	6·1	79	76
South arm	5·8	6·0	81	73
Mean values	5·8	6·2	85	82

In the theoretical treatment (as well as in the simulation experiment) it was assumed that vehicles continued at normal speed until they reached the intersection, joining the queue at times represented by A, A', etc. in Fig. 1. However, owing to the finite extension of the queue in practice, vehicles would join the

TABLE 6

COMPARISON OF THE COMPUTED VALUES OF AVERAGE QUEUE LENGTH AT THE BEGINNING OF THE GREEN PERIOD WITH THEORETICAL VALUES

Degree of saturation x	λ \ M*	2·5 Computed	2·5 Theoretical (equation (8))	10·0 Computed	10·0 Theoretical (equation (8))	40·0 Computed	40·0 Theoretical (equaton (8))
0·3	0·4	1·5	1·5†	6·0	6·0†	24·1	24·0†
	0·8	0·5	0·5†	2·1	2·0†	8·3	8·0†
0·5	0·4	1·6	1·5†	6·0	6·0†	24·2	24·0†
	0·8	0·7	0·6	2·2	2·0†	8·3	8·0†
0·7	0·4	2·1	2·0	6·2	6·0†	24·3	24·0†
	0·8	1·2	1·2	2·5	2·1	8·5	8·0†
0·8	0·4	2·7	2·7	6·7	6·5	24·5	24·0†
	0·8	1·9	1·9	3·1	2·9	8·8	8·0†
0·9	0·4	5·6	5·6	8·8	8·7	26·1	25·4
	0·8	4·8	4·8	5·3	5·2	10·6	9·8
0·95	0·4	11·2	11·2	13·1	13·1	30·1	29·8
	0·8	10·4	10·4	9·9	9·8	15·1	14·7
0·975	0·4	25·0	24·9	23·5	23·5	39·3	39·1
	0·8	23·5	24·0	20·0	20·0	24·4	24·3

* M is the average number of vehicles arriving per cycle $= qc$

† Equation (8) was read as $N = qr$ for these values

queue earlier than these times and a correction should be applied to the above formula. The corrected expression is given by equation (5·9) in Appendix 5 as

$$N = q\left(\frac{r}{2} + d\right)\left(1 + \frac{qj}{av}\right) \text{ or } qr\left(1 + \frac{qj}{av}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

whichever is the larger, where j is the average spacing of vehicles in the queue, a is the number of lanes and v is the free running speed of the traffic.

For general purposes equation (8) is adequate since the correction factor affects the results by only 5 to 10 per cent.

It is also useful to know how long the queue is likely to be in extreme cases, say, one cycle in a hundred or one cycle in twenty. Tables 7 and 8 show the possible extensions of the queue at the beginning of the green period in these infrequent cases. A probability of one in a hundred that the queue will exceed the given value means that, for a cycle of 60 seconds, say, the queue will extend beyond the given value only once in about 2 hours. These tables can be used for any fixed-time intersection.

TABLE 7

CRITICAL MAXIMUM QUEUES (1 IN 100)

(Probability of the maximum queue in any cycle being equal to or greater than the critical value is 1 per cent)

Degree of saturation x	λ \ M*	2·5	5·0	10·0	20·0	40·0
0·3	0·4	6	9	14	23	38
	0·6	5	6	11	17	28
	0·8	3	5	7	12	17
0·5	0·2	7	9	17	29	53
	0·4	6	9	14	23	38
	0·6	5	7	11	17	28
	0·8	4	5	7	12	18
0·7	0·2	9	12	17	28	50
	0·4	9	9	15	23	38
	0·6	8	9	12	18	28
	0·8	7	7	8	12	18
0·8	0·2	13	15	19	28	50
	0·4	12	13	17	24	39
	0·6	12	13	14	20	28
	0·8	11	12	12	15	18
0·9	0·2	29	25	29	38	55
	0·4	28	24	27	33	46
	0·6	27	24	26	28	42
	0·8	27	23	24	25	29
0·95	0·2	40	36	38	47	65
	0·4	40	34	37	44	55
	0·6	40	32	30	42	48
	0·8	39	32	34	36	40
0·975	0·2	82	70	79	69	93
	0·4	83	66	75	65	82
	0·6	82	70	69	58	79
	0·8	79	65	66	56	79

**M* is the average number of vehicles arriving per cycle = qc

TABLE 8

CRITICAL MAXIMUM QUEUES (1 IN 20)

(Probability of the maximum queue in any cycle being equal to or greater than the critical value is 5 per cent)

Degree of saturation x	λ \ M*	2·5	5·0	10·0	20·0	40·0
0·3	0·4	5	7	12	20	34
	0·6	4	5	9	15	24
	0·8	3	4	6	9	15
0·5	0·2	6	7	15	26	47
	0·4	5	7	12	20	35
	0·6	4	5	9	15	24
	0·8	3	4	6	9	15
0·7	0·2	7	9	15	25	44
	0·4	6	8	12	20	34
	0·6	5	7	9	15	25
	0·8	5	5	7	9	15
0·8	0·2	9	12	16	25	46
	0·4	8	11	14	21	35
	0·6	8	9	11	16	25
	0·8	7	8	9	11	16
0·9	0·2	19	18	22	30	49
	0·4	19	17	20	23	39
	0·6	19	16	17	21	34
	0·8	18	15	15	18	22
0·95	0·2	36	28	33	40	55
	0·4	35	27	30	35	47
	0·6	34	26	25	34	39
	0·8	34	25	27	27	32
0·975	0·2	74	63	65	62	84
	0·4	74	57	65	59	75
	0·6	69	61	62	54	65
	0·8	65	56	61	52	64

*M is the average number of vehicles arriving per cycle = qc

Test experiments have been carried out at three intersections in the London area. Queues were recorded at the beginning of each green period for about three hours. The mean queue length is compared in Table 5 with values calculated from equation (9). Calculated values of delay were substituted in the formula.

Table 8 was used to give the critical value of queue which should only be exceeded in 1 out of every 20 cycles on the average. It was found that the critical queue was exceeded once in 17 cycles over about 6 hours of observations.

More detailed tests were carried out at the intersection of Putney High Street and Upper Richmond Road, London, where it was found that the observed values of delay, queue and degree of saturation agreed with calculated values within the limits of scatter expected from the sizes of the samples.

Stops and starts

Proportion of vehicles which stop at least once. It is assumed for simplicity that all vehicles which arrive at the intersection while the queue of vehicles is discharging, have to stop, although in practice some of them may only have to slow down. The following expression for the proportion of vehicles which stop at least once (denoted by R) is deduced in Appendix 6:

$$R = \frac{1 - \lambda}{1 - y} \qquad \text{.......................................(10)}$$

Average number of stops and starts per vehicle. The average number of stops and starts made by vehicles is an important factor when considering the wear and tear of vehicles, fuel consumption and annoyance to drivers. The total number of stops and starts in each cycle is approximately equal to the number of vehicles in the queue at the beginning of the green period plus those vehicles which arrive while the queue is clearing. As already mentioned some of these vehicles would probably only have to slow down but for simplicity it is assumed here that they stop.

If the average queue at the beginning of the green period can clear during the green, i.e. if

$$\frac{N}{s - q} < g$$

then the average number of stops and starts (P) is shown in Appendix 6, to be:

$$P = \frac{N}{qc(1 - y)} \qquad \text{.......................................(11)}$$

where N is given approximately by equation (8).

If $\dfrac{N}{s - q} > g$ (i.e. if the queue N cannot be discharged fully during the green period)

then

$$P = \frac{N}{qc} + \lambda \quad \text{(see Appendix 6) ....(12)}$$

Figure 9, for a particular case, shows how the average number of stops and starts compares with the proportion of vehicles that stop at least once.

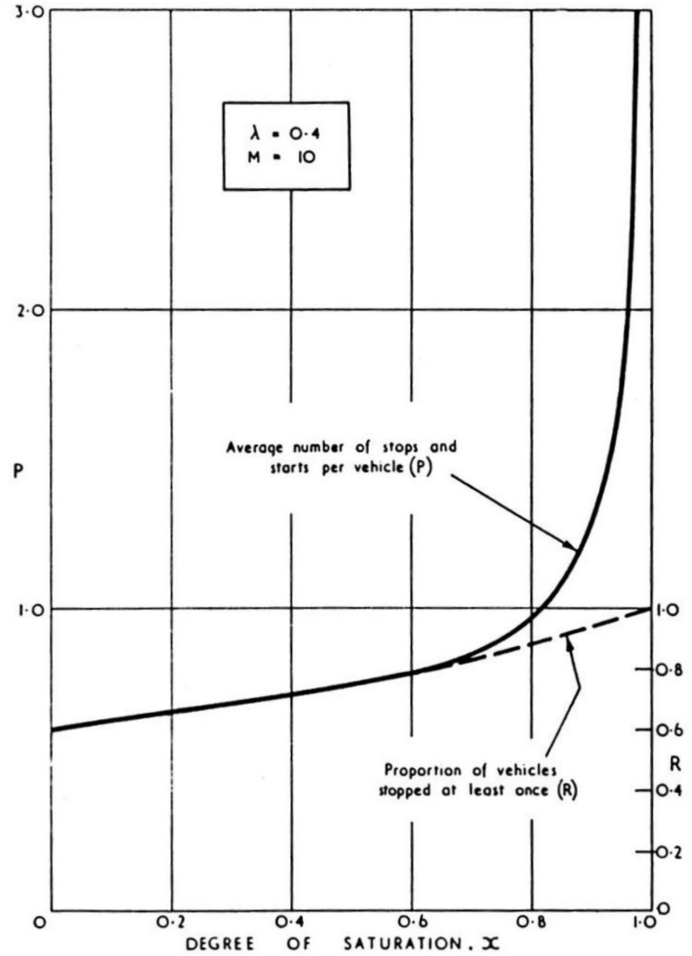

FIG. 9. COMPARISON OF THE NUMBER OF VEHICLES STOPPED AT LEAST ONCE AND THE AVERAGE NUMBER OF STOPS AND STARTS PER VEHICLE IN A PARTICULAR CASE

OBSERVATIONS ON THE ROAD

The following observations require to be taken when setting signals:

Flow

A census of all traffic using the intersection over several hours of the day (including all peak periods) is required. The census should preferably include a count of the number of right-turning* vehicles and the number of medium and heavy commercial vehicles.

* or left-turning where the right-hand rule of the road applies.

Saturation flow

Saturation flow may be measured on the road by counting the number of vehicles discharged from the queue during a representative number of fully saturated green periods and dividing the mean number by the average effective green period assuming a fixed value of lost time (say, 2 seconds). More refined methods for measuring saturation flow and lost time are available, but it has been found that the above method gives reasonable results in practice.

If in the majority of cycles the queue clears before the end of the green period the saturation flow may be obtained approximately by recording the time during which the queue is clearing and the number of vehicles discharged during this saturation period.

Though a direct measurement is obviously desirable in order to obtain reliable results it is not always practicable or indeed possible, e.g. when designing new intersections, and rules based on measurements of saturation flow carried out by the Road Research Laboratory at a large number of existing intersections can be used. To supplement these observations controlled traffic experiments were carried out off the highway on a test track. The results of these investigations into the saturation flow (s) can be expressed in terms of passenger car units (p.c.u.), with no turning traffic and with no parked vehicles present.

For approach road width (w) between 10 and 17 feet s is given by

w (ft):	10	11	12	13	14	15	16	17
s (p.c.u./h):	1675	1700	1725	1775	1875	2025	2250	2450

but for approach widths greater than 17 feet the saturation flow is given by

$$s = 145w \text{ p.c.u./h.}$$

The experiment under controlled conditions has shown that the relationship is linear up to at least 60 feet.

Effect of composition of traffic. The effect of different types of vehicles on the saturation flow is given by the following equivalents:

1 heavy or medium commercial vehicle:	$1\frac{3}{4}$ p.c.u.
1 bus:	$2\frac{1}{4}$ p.c.u.
1 tram:	$2\frac{1}{2}$ p.c.u.
1 light goods vehicle:	1 p.c.u.
1 motorcycle:	$\frac{2}{5}$ p.c.u. (estimated)
1 pedal cycle:	$\frac{1}{5}$ p.c.u.

Effect of turning traffic. The results of the experiments gave the following equivalent:

1 right turning vehicle: $1\frac{3}{4}$ straight ahead vehicles.

This result may tend to underestimate the effect of a right-turning vehicle because cases where right-turners were given special phases or where they frequently blocked the intersection were omitted.

In nearly all cases the saturation flow/time curve (as in Fig. 2) showed that saturation flow was reasonably constant (after allowing for starting delays, etc.). However, at some sites where right-turning vehicles have been a particular problem the saturation flow has been observed to fall after the first 10 seconds or so of green time.

Effect of a parked vehicle. It has been found that the reduction in saturation flow caused by a parked vehicle is equivalent to a loss of carriageway width at the stop line. The experiments showed that with green periods of the order of 25 seconds a car parked at the stop line had the same effect on saturation flow as if the road were $5\frac{1}{2}$ feet narrower; at clear distances of 50, 100 and 150 feet from the stop line the loss in saturation flow was the same as if the road was respectively $4\frac{1}{2}$, $2\frac{1}{2}$ and 1 foot narrower. Allowing for the theoretical effect of changing the green period, this can be expressed approximately by

$$\text{effective loss of carriageway width} = 5{\cdot}5 - \frac{0{\cdot}9(z-25)}{k} \text{ feet} \qquad \ldots(13)$$

where z = clear distance of parked car from stop line (feet) and k = green time (seconds).

If $z < 25$ feet the second term should be taken as zero and if the second term is greater than 5·5 feet the whole expression should be taken as zero.

The effective loss should be increased by 50 per cent for a parked lorry or wide van.

SUMMARY OF PROCEDURE FOR SETTING TRAFFIC SIGNALS

In a period where the traffic flow is varying randomly about the mean, the procedure for obtaining optimum settings is as follows:

(i) Estimate the flow and saturation flow for each arm of the intersection.

(ii) Evaluate the ratio of flow to saturation flow for each arm, and select the y value for each phase (i.e. the maximum q/s value).

(iii) Add the y values together to give Y for the whole intersection.

(iv) Decide on all-red periods for pedestrians, turning traffic, etc. and estimate the lost time, R, due to this, e.g. if sequent ambers occur twice per cycle then $R = 6$ seconds; if there are two all-red periods of 2 seconds each then $R = 10$ seconds (see Fig. 6).

(v) Calculate the cycle time from equation (4):

$$c_0 = \frac{1{\cdot}5L+5}{1-Y}$$

where L is the total lost time per cycle, given by

$$L = nl+R$$

where n is the number of phases and l is the average lost time per phase due to starting delays.

(vi) Subtract the total lost time, L, from the cycle time giving the available green time and divide this in the ratio of the y values, i.e.

$$g_1 = \frac{y_1}{Y}(c_0-L)$$

$$g_2 = \frac{y_2}{Y}(c_0-L) \text{ etc.}$$

(vii) Add l seconds to each effective green time, g_1, g_2, . . . and subtract the amber period (3 seconds) to give the controller setting of green time.

SUMMARY

Delays at intersections controlled by traffic signals have been investigated using an electronic computing machine to simulate traffic conditions.

A formula for the average delay per vehicle on a single approach to an intersection controlled by fixed-time traffic signals (or vehicle-actuated signals working on a fixed cycle because of heavy traffic demands) has been derived from the computed results.

Formulae have been deduced for the cycle time and green times which give the least delay to all vehicles using the intersection. Tables and formulae for queues and the number of stops and starts of vehicles have been obtained. These formulae and tables have been tested under actual road conditions with satisfactory results.

ACKNOWLEDGEMENTS

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APPENDIX 1

GLOSSARY AND SYMBOLS

n Number of phases, where a phase is a state of the signals during which a particular traffic stream or group of streams receives right of way.

d Average delay per vehicle on a single approach to an intersection. It is the difference between the average journey time through the intersection and the time for a run which is not stopped or slowed down by the signals.

D Total delay per unit time for the whole intersection, i.e. flow × average delay per vehicle.

d Average delay to all vehicles using the intersection.

c Cycle time.

c_0 Optimum cycle time—the cycle time which gives the least average delay to all vehicles using the intersection.

c_m Minimum cycle time—the cycle which is theoretically just long enough to pass the traffic through the intersection.

R All-red period—the time during each cycle when all signals display red or red with amber.

l Lost time for a single phase—the amount of time in a cycle which is effectively lost to traffic movement in the phase because of starting delays and the falling-off of the discharge rate which occurs during the amber period.

L Total lost time per cycle—the sum of the lost times for each phase and those periods when all signals show red or red with amber. It is given by $(nl+R)$.

k Green time setting on the controller.

g Effective green time—the sum of the green period and the amber period less the lost time for the particular phase.

r Red time—time during which the signal is red or red with amber on a particular phase.

λ The proportion of the cycle which is effectively green for a particular phase (g/c).

q The flow—average number of vehicles passing a given point on the road in the same direction per unit of time.

Q Total flow through the intersection.

s Saturation flow—maximum rate of discharge of the queue during the green period.

y The maximum ratio of flow to saturation flow (q/s) for a given phase.

Y Summation for the whole intersection of the y values corresponding to each phase.

x Degree of saturation—ratio of the flow to the maximum flow which can just be passed through the intersection from the particular approach. It is given by $q/\lambda s$.

x_0 Degree of saturation when the cycle time and green times have optimum values.

N Average queue at the beginning of the green period.

M Average number of vehicles arriving in each cycle.

P Average number of times that a vehicle is stopped on a particular approach to an intersection.

R The proportion of vehicles which are stopped at least once.

APPENDIX 2

SIMULATION METHOD USED FOR ESTIMATING DELAYS

A SIMULATION method is one in which actual events are reproduced using a 'model' and information is obtained from the behaviour of the model instead of from the events themselves. Such a method is often used where the actual events are too complicated to study empirically owing to the number of variables involved or where an analytical solution is too difficult.

Simulation can be carried out by making a special model to suit the particular purpose or, if the problem can be reduced to a series of logical manipulations (which, if the amount of work involved were not prohibitive, could be carried out manually), by using a general purpose digital computing machine. In the case of the traffic signal problem the Pilot Model of the ACE* was used. Since this is a general purpose machine, a theoretical 'model' for estimating delays at fixed-time signals was formulated based on the following assumptions:

(i) For fixed-time signals each approach to an intersection can be studied independently.

(ii) Traffic is assumed to arrive at the intersection at random. In fact, the actual distribution obtained from observations on the road could be used but random traffic has the advantage that it can be generated artificially using tables of random numbers to derive the intervals between successive vehicles.

(iii) Vehicles are discharged from the queue at a constant rate (called the saturation flow) during the effective green time. During the red time no flow takes place. The first vehicle in the green time to be discharged is delayed a 'random' fraction of a normal discharge interval in order that the number of vehicles discharged in fully saturated green periods will be proportional to the length of the effective green time. This provides continuity when investigating the effect of different green times.

(iv) Delay is defined more clearly by referring to Fig. 1, where the position of a vehicle along a road is plotted against time. The 'in' point shown in Fig. 1 is a point just outside the influence of the intersection and the 'out' point is at such a distance that a vehicle, after passing through the intersection would have attained normal running speed. Delay is defined as the difference between the time taken to travel through the intersection from the 'in' point to the 'out' point and the time taken to travel that distance at the normal running speed (i.e. if the intersection were not there). For any particular vehicle, say X, it will be seen that the delay is given by AB.

* The Pilot Model ACE (Automatic Computing Engine) was a high-speed electronic calculating machine housed at the National Physical Laboratory, Teddington, England. It has now been superseded by the DEUCE (Digital Electronic Universal Computing Engine).

In the theoretical model some simplicity can be introduced without affecting the accuracy of the delay measurement. It can be assumed that vehicle X (Fig. 1) arrives at the stop line at A and departs at B. Similarly vehicle X′ arrives at A′ and departs at B′. The time distribution of the arrivals A, A′, etc. will be the same as that at the 'in' point so that if the latter is assumed to be random then the distribution of A, A′, etc, is also random. Similarly, the distribution of B, B′, etc. is the same as that at the 'out' point. Thus, once the lost time has expired vehicles in the model can be discharged at regular intervals.

The Pilot ACE receives all instructions and data on Hollerith cards and traffic was therefore prepared in this form by punching holes in the cards to represent the arrival of vehicles at the intersection. In this investigation time is the fundamental variable and a quantized time scale was obtained by considering each position on the card in order as one unit of time. Each card of the pack extends the time scale by a given number of units. A hole punched in a particular position on a particular card means that a vehicle has arrived at the intersection in that unit of time. The shorter the unit of time, the nearer the time scale approaches a continuous one.

Traffic was generated using a sequence of random numbers (from published tables) to decide whether a vehicle arrives in each successive unit.

Example

It is desired to generate traffic of 720 vehicles/hour.

The unit of time on the cards is chosen to be ½ second.

The average rate of arrival is therefore 0·1 vehicle per unit of time.

A number is taken from a table of random numbers for each unit of time and interpreted as a decimal fraction; if it is less than 0·1 an arrival is assumed in the corresponding unit of time; if it is greater than 0·1 there is no arrival in that unit.

In practice, the method was rather more refined than described above, obtaining the maximum possible use from each random number[8].

Since this method does not allow two or more vehicles to arrive in the same unit of time, the unit should be chosen as small as practicable so that the departure from reality is not important.

The computer is 'programmed' to:

(a) Interpret the traffic cards.

(b) Act as the traffic signal by timing off alternate red and green periods.

(c) Keep a count of the queue, adding one for each arrival and subtracting one at constant intervals during the green time to represent the discharge of vehicles, until the queue becomes zero.

(d) Compute the total delay experienced by all vehicles (see below).

The total delay is computed by adding the number of vehicles in the queue into a storage counter every unit of time, e.g. if, in successive units of time, the number of vehicles in the queue is 2, 3, 3, 2, 3 and the unit of time is ½ second then at the end of the first half second the total delay experienced is $2\times\frac{1}{2}$ vehicle-second, at the end of the second unit of time the total delay is $(2\times\frac{1}{2}+3\times\frac{1}{2})$ vehicle-seconds and at the end of the fifth unit of time the total delay is $(2+3+3+2+3)\,\frac{1}{2}$ vehicle-seconds. If n is the queue during any unit of time and u is the value of the unit then the total delay is $u\Sigma n$ and the average delay per vehicle is obtained by dividing this by the number of arrivals. Because of the high operating speed of the ACE the delay to about 10 000 vehicles can be computed in 5 minutes, i.e. for a flow of 1000 vehicles/hour, 10 hours' traffic can be analysed in 5 minutes.

APPENDIX 3

OPTIMUM CYCLE TIMES

n-phase intersection

For a particular intersection, optimum conditions are obtained by minimizing the total delay with respect both to cycle time and to the division of the cycle. Calculations of the average delay per vehicle have shown that the least delay is obtained for a given cycle time when the effective green times of the phases are approximately in proportion to the y values of the phases. The effect of cycle time on delay will now be investigated.

The total delay for each arm of the intersection per unit of time is the product of the average delay per vehicle and the flow. Thus, the total delay for the intersection as a whole is given by

$D = \Sigma$ (Average delay per vehicle) $\times$ flow

where the average delay per vehicle is given approximately by equation (2). Rearranging, we have

$$D = \sum_{1}^{n} \left(\frac{c y_r s_r (1-\lambda_r)^2}{2(1-y_r)} + \frac{y_r^2}{2\lambda_r(\lambda_r - y_r)} \right) \quad \ldots\ldots\ldots\ldots\ldots(3{\cdot}1)$$

Differentiating with respect to the cycle time gives

$$\frac{dD}{dc} = \sum_{1}^{n} \left\{ \frac{(1-\lambda_r)^2 y_r s_r}{2(1-y_r)} - \frac{d\lambda_r}{dc} \left(\frac{y_r^2(2\lambda_r - y_r)}{2\lambda_r^2(\lambda_r - y_r)^2} + \frac{c y_r s_r (1-\lambda_r)}{(1-y_r)} \right) \right\} \quad \ldots(3{\cdot}2)$$

$= 0$ for minimum delay,

i.e. $$\sum_{1}^{n} \frac{y_r s_r (1-\lambda_r)}{1-y_r} \left(\frac{1-\lambda_r}{2} - \frac{c_0 d\lambda_r}{dc} \right) - \sum_{1}^{n} \frac{y_r^2 (2\lambda_r - y_r)}{2\lambda_r^2 (\lambda_r - y_r)^2} \cdot \frac{d\lambda_r}{dc} = 0 \quad \ldots(3{\cdot}3)$$

If λ is made proportional to y, then $\lambda_r = \frac{c-L}{c}\frac{y_r}{Y}$ and $\frac{d\lambda_r}{dc} = \frac{y_r L}{Yc^2}$ where L is the lost time per cycle.

Substituting for λ_r and $\frac{d\lambda_r}{dc}$ in equation (3·3) we have

$$\frac{1}{Y}\sum_{1}^{n} \frac{y_r s_r}{1-y_r}(c_0^2(Y-y_r)^2 - L^2 y_r^2) - L \sum_{1}^{n} \frac{y_r^3(2\lambda_r - y_r)}{\lambda_r^2(\lambda_r - y_r)^2} = 0 \quad \ldots\ldots(3{\cdot}4)$$

Therefore

$$\frac{1}{Y}\sum_{1}^{n}\frac{y_r s_r}{1-y_r}\left\{c_0^2(Y-y_r)^2-L^2y_r^2\right\}\left\{(c_0^2-2c_0L)+L^2\right\}-$$

$$-LY^3c_0^3n\frac{[c_0(2-Y)-2L]}{[c_0(1-Y)-L]^2}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(3\cdot5)$$

In the above expression the last term in each brace is for most practical purposes very small in comparison with the first term and an approximation for them seems justifiable. When calculated values of delay were plotted against cycle time for several intersections it was found that the optimum cycle was approximately equal to twice the minimum cycle, shown by the vertical asymptote to each curve (see Fig. 7). The minimum cycle will now be derived.

The minimum cycle, c_m, is just long enough to allow all the traffic which arrives in one cycle (assuming uniform flow) to pass through the intersection in the same cycle. With random traffic, however, any green time which is wasted because of the variability of the arrival times can never be recovered and the minimum cycle in consequence is associated with very high delays (theoretically, with infinite delays). It is the sum of the lost time per cycle and the amount of time necessary to pass all the traffic through the intersection at the maximum possible rate, i.e.

$$c_m = L+\frac{q_1}{s_1}c_m+\frac{q_2}{s_2}c_m+\ldots+\frac{q_n}{s_n}c_m$$

where q_n/s_n is the highest ratio of flow to saturation flow for the nth phase. Thus

$$c_m = L+c_m(y_1+y_2+\ldots+y_n)$$
$$= L+c_mY$$

and $c_m = \dfrac{L}{1-Y}$

The optimum cycle is therefore, approximately, $2L/(1-Y)$ and we may replace L by $\dfrac{c_0(1-Y)}{2}$ in those terms which are comparatively small.

The term in the first brace becomes

$$c_0^2\left((Y-y_r)^2-\frac{y_r^2}{4}(1-Y)^2\right)$$

and that in the second brace becomes

$$(c_0^2-2c_0L)\left\{1+\frac{L^2}{c_0^2-2c_0L}\right\}$$

$$=(c_0^2-2c_0L)\left\{1+\frac{(L/c_0)^2}{(1-2L/c_0)}\right\}$$

$$=c_0(c_0-2L)\frac{(1+Y)^2}{4Y}$$

Equation (3·5) now becomes

$$\frac{(1+Y)^2}{16Y^5n}L(c_0-2L)\sum_{1}^{n}\frac{y_rs_r}{1-y_r}\{4(Y-y_r)^2-y_r^2(1-Y)^2\}-$$

$$-\frac{L^2[c_0(2-Y)-2L]}{[c_0(1-Y)-L]^2}=0 \qquad \text{.....................(3·6)}$$

$$\text{Let } E\equiv\frac{L(1+Y)^2}{16nY^5}\sum_{1}^{n}\frac{y_rs_r}{1-y_r}\{4(Y-y_r)^2-y_r^2(1-Y)^2\} \qquad \text{..................(3·7)}$$

The equation reduces to

$$(c_0-2L)E-\frac{L^2[c_0(2-Y)-2L]}{[c_0(1-Y)-L]^2}=0 \qquad \text{................(3·8)}$$

Thus

$$c_0^3(1-Y)^2-2c_0^2L(1-Y)(2-Y)+c_0L^2\left[5-4Y-\frac{(2-Y)}{E}\right]+$$

$$+L^3\left[\frac{2}{E}-2\right]=0 \qquad \text{.......................(3·9)}$$

Since the optimum cycle has been found to be approximately $2L/(1-Y)$ in several cases, let us now assume it is given accurately by

$$c_0=\frac{2L}{1-Y}F \qquad \text{........................(3·10)}$$

where F is a factor depending on the flows, saturation flows and the lost time of the intersection. We may combine the last two terms of equation (3·9) by replacing L^3 (in the last term) by $L^2.\frac{c_0(1-Y)}{2F}$. Dividing throughout by c_0 and solving for a quadratic we have

$$c_0=\frac{2L}{1-Y}\left\{1+\frac{\sqrt{Y^2-Y/F+1/E+[1/F-1][1-(1-Y)/E]}-Y}{2}\right\} \qquad \text{..(3·11)}$$

For most practical purposes F is sufficiently close to unity for equation (3·11) to be written as

$$c_0=\frac{2L}{1-Y}\left\{1+\frac{\sqrt{Y^2-Y+1/E}-Y}{2}\right\} \qquad \text{............(3·12)}$$

so that F in equation (3·10) will be given to a first approximation by

$$F=\left\{1+\frac{\sqrt{Y^2-Y+1/E}-Y}{2}\right\} \qquad \text{................(3·13)}$$

When determining the optimum cycle length, F may be evaluated from equation (3·13) and its value substituted in equation (3·10) to give the cycle time. If, however, the value so found differs by more than about 10 per cent from unity then equation (3·11) should be used to obtain a more accurate answer, knowing the approximate value of F.

Practical determination of the optimum cycle

To allow the cycle time to be evaluated more easily the equations can be broken down and converted into graphical form as follows:

Rearrange equation (3·7) to make E a function of (Gy_r) instead of y_r, where

$$G = \frac{3-Y}{2Y} \qquad \text{......................(3·14)}$$

Thus

$$E = \frac{L}{Y^2 n} \sum_1^n s_r(Gy_r)[1-(Gy_r)]\frac{1}{G}\left\{\frac{(1+Y)^2}{4Y} \times \frac{1-(Gy_r)[(1+Y)/(3-Y)]}{1-(Gy_r)[2Y/(3-Y)]}\right\} \qquad (3·15)$$

It has been found that a one per cent error in E produces only approximately one-fifth per cent error in the optimum cycle length over the range of practical interest. The value of E need not therefore be known too accurately and equation (3.15) may be simplified by replacing the brace term of this equation by unity. This term is approximately unity for most values of y_r and Y but falls to about 0·8 when y_r forms a large proportion of Y. However, if one phase has a relatively large y value the other phases must have small values (where the approximation has little effect) and the overall error in E will be considerably less than the term containing y_r as a large proportion of Y. The maximum error in the optimum cycle due to this approximation is 4 to 5 per cent in extreme cases. The normal error is likely to be less than 2 per cent.

Thus,

$$E = \frac{L}{Y^2 n} \sum_1^n s_r \frac{1}{G}[Gy_r(1-Gy_r)] \qquad \text{..............(3·16)}$$

Let

$$E = \frac{L}{Y^2 nG} \sum_1^n s_r B_r \text{ say,} \qquad \text{..................(3·17)}$$

where

$$B_r = Gy_r(1-Gy_r) \qquad \text{....................(3·18)}$$

The expression for B_r has been plotted in terms of Gy_r (see Fig. 10). Since E involves the summation term its value will have to be determined by calculation, and hence, the equation should be made as simple as possible. Let us define a new quantity Z such that

$$Z = \frac{1}{EY^2 G} \qquad \text{......................(3·19)}$$

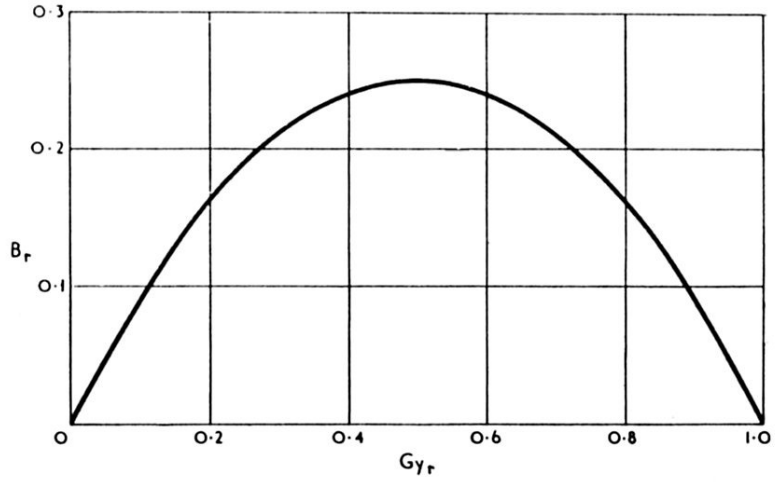

FIG. 10. THE OPTIMUM CYCLE—FACTOR B_r

From equation (3·17), Z may be given by

$$Z = \frac{n}{L \sum_1^n s_r B_r} \quad \ldots\ldots (3\cdot20)$$

Substituting for E in equation (3·13), we have

$$F = \quad 1 + \frac{\sqrt{Y^2 - Y + Y(3-Y)Z/2} - Y}{2} \quad \ldots\ldots (3\cdot21)$$

Similarly, equation (3·11) can be modified to give

$$F_{\text{corrected}} = 1 + \frac{\sqrt{Y^2 - Y/F + Y(3-Y)Z/2 + [1/F - 1][1-(1-Y)(3-Y)YZ/2]} - Y}{2} \quad (3\cdot22)$$

Values of F, over the whole practical range of Y and Z, were obtained to a first approximation from equation (3·21). The values obtained were substituted in equation (3·22) to find a second approximation to the true F values. A family of curves of F against Z for different Y values showed that the Y=0·5 curve had the highest values, all the other curves lying below this one. Since it is perhaps better to have the cycle longer than the optimum value rather than shorter, only the Y=0·5 curve is shown in Fig. 11. For values of Y between 0·5 and 1·0 the F values lie within a range of 0·05, thus, it was not considered necessary to complicate the graph with these curves.

The determination of the optimum cycle is thus reduced to the following simple steps:

(i) Work out $G=(3 - Y)/2Y$, and multpily each y value by G, giving Gy_r for each phase.

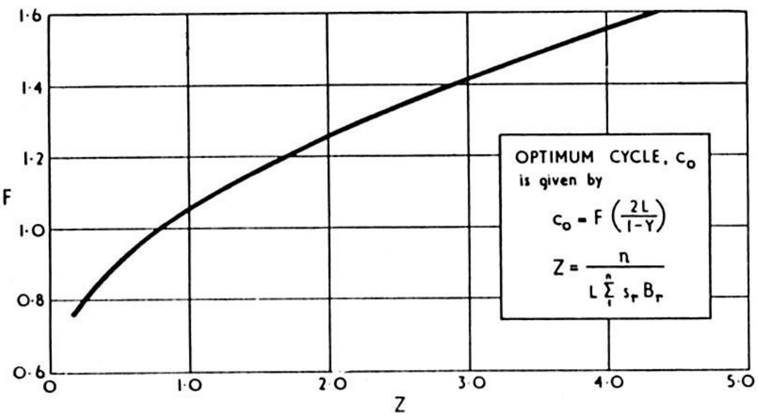

FIG. 11. THE OPTIMUM CYCLE—FACTOR F

(ii) From Fig. 10 read the value of B_r corresponding to the particular value of Gy_r for each phase and multiply each B_r by the appropriate value of the saturation flow. Carry out the summation for all the phases, giving

$$\sum_1^n s_r B_r$$

(iii) Now substitute in equation (3·20), namely

$$Z = \frac{n}{L\sum_1^n s_r B_r}$$

to find Z for the whole intersection, where n is the number of phases and L is the total lost time per cycle in the same units as s^{-1}.

(iv) From Fig. 11 read the value of F corresponding to the Z value and substitute in equation (3·10) to give the optimum cycle length

$$c_0 = F.\frac{2L}{1-Y}$$

Approximate formula for the optimum cycle

Although the individual steps in the method just given for determining the optimum cycle are relatively simple the method as a whole is perhaps too elaborate for many purposes. On the other hand, the very rough empirical formula, $c_0 = \frac{2L}{1-Y}$, given earlier, may be too inaccurate. It is felt that something between the two methods is required, i.e. a simple formula which has a wider application than the above formula. Use was made of the method just described to derive such a formula.

A 2-phase intersection was first considered with y values of the phases in the ratio of (a) 1:1, (b) 2:1 and (c) 3:1*. Three values of average saturation flow of the intersection, 1200, 1800, 3600 vehicles/hour, were considered. Such a range of y values and saturation flows covers most practical cases. Values of Z were calculated from equation (3·20) for a lost time of 10 seconds and corresponding values of F were obtained from Fig. 11. These values are shown in Table 9 where it can be seen that F varies from 0·87 to 1·24.

TABLE 9

FACTOR F WITH A LOST TIME OF 10 SECONDS

Average saturation flow (vehicles/hour) \ y ratio	1:1	2:1	3:1
1200	1·11	1·17	1·24
1800	1·01	1·05	1·11
3600	0·87	0·91	0·95

Having established practical limits for the factor F (and hence the optimum cycle time) the variation with respect to lost time was then considered in the two extreme cases in Table 9 and in one intermediate case:

(1) $s =$ 1200 vehicles/hour, y ratio 3:1

(2) $s =$ 1800 vehicles/hour, y ratio 2:1

(3) $s =$ 3600 vehicles/hour, y ratio 1:1

F is found as before from equation (3·20) and Fig. 11. The product $2LF$, i.e. the numerator in the cycle time formula, is formed and is shown in Fig. 12 plotted against lost time (solid lines). It was found that a linear approximation to these curves could be obtained from a formula of the type

$$c_0 = \frac{KL + 5}{1 - Y} \text{ seconds} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3{\cdot}23)$$

where K=1·98, 1·60, 1·24, respectively, in the three cases. These are shown as dotted lines in Fig. 12. The agreement is quite good especially since the lost time is not likely to be less than about 4 seconds anyway.

K values have also been determined for other values of saturation flow and y ratios using equation (3·20) and Fig. 11. The values are shown in Table 10.

The table covers nearly all cases of 2-phase intersections likely to occur in practice. However, a single value of K is required which is representative of the more common junctions. Some of the junctions covered in Table 10 are quite rare, e.g. a y ratio of 3:1. In selecting an average case the most common junctions should receive special consideration, e.g. those with y ratios between 1:1 and 2:1

* Preliminary calculations showed that the numerator of the optimum cycle formula, $2LF$, was not appreciably affected by changes in the Y value of the intersection (assuming constant y ratio) over the usual working range of Y: 0·4—0·9.

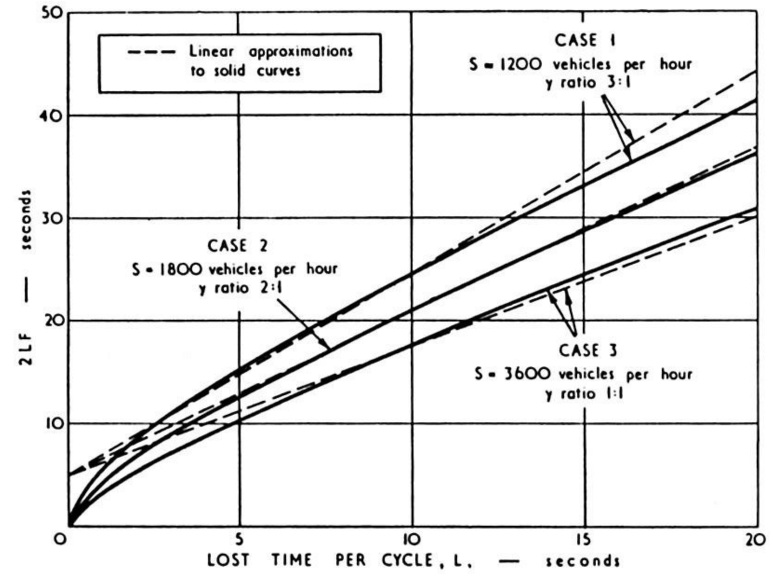

FIG. 12. THE VARIATION OF THE NUMERATOR OF THE OPTIMUM CYCLE TIME FORMULA WITH LOST TIME

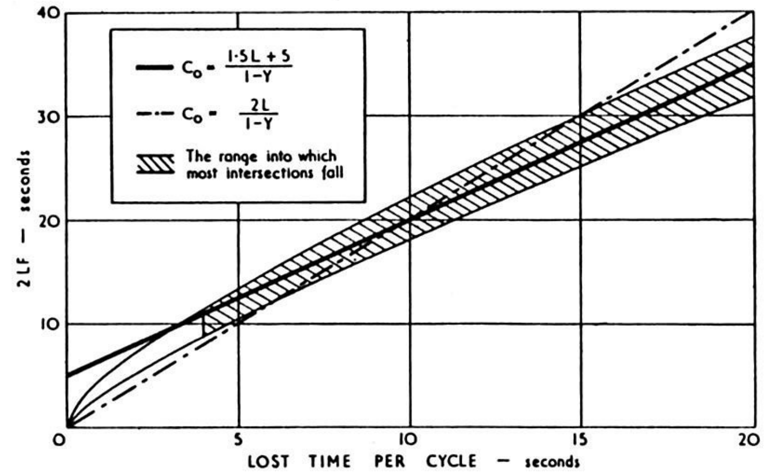

FIG. 13. LINEAR APPROXIMATIONS TO NUMERATOR OF OPTIMUM CYCLE FORMULA

and average saturation flows between 1500 and 3000 vehicles/hour. The K values for this range (see Table 10) lie between 1·31 and 1·70. Thus, a typical junction might have $K=1{\cdot}50$. Calculations of $2LF$ were made for these limits and these are shown in Fig. 13, together with curves derived from the simple equation, $c_0 = \frac{2L}{1-Y}$, and from $c_0 = \frac{1{\cdot}5L+5}{1-Y}$ seconds. It can be seen that the latter formula fits the calculated results better than the original simple formula.

TABLE 10

VALUES OF K IN THE FORMULA $c_0 = \frac{KL+5}{1-Y}$ seconds

Average saturation flow (vehicles/hour) \ y ratio	1·1	2:1	3:1
1200	1·72	1·84	1·98
1500	1·60	1·70	1·84
1800	1·52	1·60	1·72
2100	1·46	1·54	1·64
2400	1·40	1·48	1·56
3000	1·31	1·39	1·48
3600	1·24	1·32	1·40

Multiphase intersections

Calculations of Z (equation (3·20)) for a number of symmetrical intersections (y ratio 1:1) showed that Z was roughly independent of the number of phases, for a given lost time. For asymmetrical multiphase intersections values of Z were within the range of variation obtained for 2-phase intersections. Although it would be difficult to construct a complete table such as Table 10 for values of K for multiphase junctions, the simple formula derived for the average 2-phase junction can be extended to include average multiphase junctions. Thus, in all cases

$$c_0 = \frac{1{\cdot}5L + 5}{1 - Y} \text{ seconds, approximately.} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3{\cdot}24)$$

APPENDIX 4

OPTIMUM CONDITIONS AT AN INTERSECTION

Degree of saturation

When an intersection which is controlled by fixed-time traffic signals is working under optimum conditions the cycle time is given to a first approximation by

$$c_0 = \frac{2L}{1 - Y} \quad \text{.......................................(4·1)}$$

and the proportions of green time for n phases satisfy

$$\lambda_1 + \lambda_2 + \ldots + \lambda_r + \ldots + \lambda_n = \frac{c_0 - L}{c_0} \quad \text{..............(4·2)}$$

$$= 1 - \frac{L}{c_0}$$

From equation (4·1) $\dfrac{L}{c_0} = \dfrac{1 - Y}{2}$ ....(4·3)

Thus

$$\sum_1^n \lambda_r = \frac{1}{2}(1 + Y) \quad \text{....................................(4·4)}$$

Since the green times are approximately proportional to the y values,

$$\frac{\lambda_r}{\lambda_{r+1}} = \frac{Y_r}{Y_{r+1}}$$

Rearranging, we have

$$\frac{Y_r}{\lambda_r} = \frac{Y_{r+1}}{\lambda_{r+1}}$$

i.e. $x_r = x_{r+1} = x_0$ say ....(4·5)

Thus, the degree of saturation is the same for all phases when the intersection is working under optimum conditions. Now, $\lambda_1 = \dfrac{y_1}{x_0}$, $\lambda_2 = \dfrac{y_2}{x_0}$, .. and so on, so that

$$\sum_1^n \lambda_r = \left(\sum_1^n y_r\right) \Big/ x_0 = \frac{Y}{x_0} \quad \text{............................(4·6)}$$

Combining with equation (4·4) we have

$$\frac{Y}{x_0} = \frac{1}{2}(1+Y)$$

Therefore

$$x_0 = \frac{2Y}{1+Y} \quad \text{.................(4·7)}$$

Average delay for the whole intersection

This derivation applies only to intersections where all arms of the same phase have approximately the same value of the ratio, flow/saturation flow.

The total delay per unit time for all arms will be given by

$$D = \sum_{1}^{n'} \left\{ \frac{c_0(1-\lambda_r)^2 q_r}{2(1-y_r)} + \frac{x_0^2}{2(1-x_0)} \right\} \text{ approximately} \qquad \ldots\ldots\ldots(4{\cdot}8)$$

where c_0 and x_0 are optimum values and n' is the number of approaches to the intersection. The summation may be represented by $Q\bar{d}$, where Q is the total flow per unit time through the intersection and $\bar{d}$ is the average delay for all vehicles. Thus,

$$Q\bar{d} = \frac{c_0}{2} \sum_{1}^{n'} \frac{(1-\lambda_r)^2 q_r}{(1-y_r)} + \frac{n' x_0^2}{2(1-x_0)} \qquad \ldots\ldots\ldots\ldots\ldots(4{\cdot}9)$$

Under optimum conditions

$$x_0 = \frac{2Y}{1+Y} \text{ (equation (4.7))}$$

$$\text{and } \lambda_r = y_r \frac{(1+Y)}{2Y}$$

Therefore,

$$Q\bar{d} = \frac{c_0}{2} \sum_{1}^{n'} \frac{\left(1 - \frac{1+Y}{2Y} y_r\right)^2 q_r}{(1-y_r)} + \frac{2n' Y^2}{(1-Y^2)} \qquad \ldots\ldots\ldots\ldots(4{\cdot}10)$$

$$= \frac{c_0}{2} \sum_{1}^{n'} \left(1 - \frac{1+Y}{2Y} y_r\right)^2 (1 + y_r + y_r^2) q_r + \frac{2n' Y^2}{(1-Y^2)}$$

$$= \frac{c_0}{2} \sum_{1}^{n'} \left\{1 + y_r\left(1 - \frac{1+Y}{Y}\right) + y_r^2\left(\left[\frac{1+Y}{2Y}\right]^2 + 1 - \frac{1+Y}{Y}\right)\right\} q_r + \frac{2n' Y^2}{(1-Y^2)}$$

To a first approximation we may neglect terms in y_r^2 giving

$$Q\bar{d} = \frac{c_0}{2} \sum_{1}^{n'} \left(1 - \frac{y_r}{Y}\right) q_r + \frac{2n' Y^2}{(1-Y^2)}$$

$$= \frac{c_0}{2}\left(Q - \frac{\sum_{1}^{n'} y_r q_r}{Y}\right) + \frac{2n' Y^2}{(1-Y^2)}$$

Thus

$$\bar{d} = \frac{c_0}{2}\left(1 - \frac{\sum_{1}^{n'} y_r q_r}{YQ}\right) + \frac{2n' Y^2}{Q(1-Y^2)}$$

$$= \frac{c_0}{2}\left\{1 - \frac{\sum_{1}^{n'} y_r q_r}{YQ} + \frac{2n' Y^2}{Q(1+Y)} \cdot \frac{2}{c_0(1-Y)}\right\} \qquad \ldots\ldots(4{\cdot}11)$$

Since $\frac{1}{L} = \frac{2}{c_0(1-Y)}$ from equation (4·1), we may write

$$\bar{d} = \frac{c_0}{2}\left\{1 - \frac{\sum_{1}^{n'} y_r q_r}{YQ} + \frac{2n'Y^2}{LQ(1+Y)}\right\} \quad \ldots\ldots\ldots\ldots\ldots(4{\cdot}12)$$

Some examples. Let us assume that the lost time is 5 seconds per phase and the saturation flow is 1800 vehicles/hour, i.e. ½ vehicle/second, on all arms.

Type	*Ratio of y values*	*Average delay expression*
2-phase	1:1	$\bar{d} = 0{\cdot}25c_0\left(\frac{1+2{\cdot}6Y}{1+Y}\right)$ ....(4·13)
„	2:1	$\bar{d} = 0{\cdot}22c_0\left(\frac{1+2{\cdot}8Y}{1+Y}\right)$ ....(4·14)
„	3:1	$\bar{d} = 0{\cdot}19c_0\left(\frac{1+3{\cdot}1Y}{1+Y}\right)$ ....(4·15)
„	4:1	$\bar{d} = 0{\cdot}16c_0\left(\frac{1+3{\cdot}5Y}{1+Y}\right)$ ....(4·16)
3-phase	1:1:1	$\bar{d} = 0{\cdot}33c_0\left(\frac{1+2{\cdot}2Y}{1+Y}\right)$ ....(4·17)
4-phase	1:1:1:1	$\bar{d} = 0{\cdot}38c_0\left(\frac{1+2{\cdot}1Y}{1+Y}\right)$ ....(4·18)

It should be noted that the above expressions for delay do not take into account the empirical correction term of equation (1). The expressions may be approximately corrected by subtracting 10 per cent of the delay.

APPENDIX 5

AVERAGE QUEUE AT THE BEGINNING OF THE GREEN PERIOD

LET us consider an arm of an intersection where the initial queue at the beginning of the green period (and reinforced by fresh arrivals during the green) disappears before the commencement of the red signal. The cycle in this case is said to be unsaturated ,and the initial queue at the beginning of the green period is simply equal to the number of vehicles which have arrived during the red, i.e.,

$$N_u = qr \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5{\cdot}1)$$

If we now consider a case in which the cycles are fully saturated over an interval of time, then the length of the queue will have gradual variations over this interval, on which are superimposed sharper increases and decreases corresponding to the red and the green periods respectively. The range of these short period fluctuations on the average will be equal to qr, and the average queue over the whole interval, assuming the queue at the end to be roughly the same as it was at the beginning, is equal to the product of the flow and the average delay per vehicle.

Thus, the average queue at the beginning of the green period is equal to the average queue throughout the interval plus half the average range of the cyclic fluctuations, i.e.

$$N_s = qd + \tfrac{1}{2}\,qr \qquad \text{(5·2)}$$

If the average queue for the group of saturated cycles decreases until the cycles are only just saturated, then its value will be $\frac{1}{2}\,qr$ and the average queue at the beginning of the green period will be qr, the same as for the unsaturated cycles. It cannot be less than this value.

The two cases considered are rather specialized, assuming that the cycles are all saturated or all unsaturated. In practice, at most intersections, there is a mixture of saturated and unsaturated cycles owing to the random nature of the flow of traffic (see Fig. 8).

For these in-between cases we may divide the time into, say, n fully saturated cycles and m unsaturated cycles. Thus, the flow q, with average delay per vehicle d, over $(m + n)$ cycles may be considered as a flow of q_s, with delay d_s, for n cycles and flow q_u, with delay d_u, for m cycles.

During n cycles the average maximum queue will be

$$\tfrac{1}{2}\,q_s r + q_s d_s$$

During m cycles the average maximum queue will be

$$q_u r$$

Therefore, the average queue at the beginning of the green period over $(m + n)$ cycles will be:

$$N = \frac{1}{m+n}\left(\tfrac{1}{2}\,n\,q_s r + n\,q_s\,d_s + m\,q_u r\right) \qquad \text{(5·3)}$$

Now the average flow is given by

$$q = \frac{n\,q_s + m\,q_u}{(n+m)}$$

and the average delay by

$$d = \frac{n\,d_s\,q_s + m\,d_u\,q_u}{(n+m)\,q}$$

Thus,

$$N = \tfrac{1}{2}\,qr + \frac{n\,q_s\,d_s + \tfrac{1}{2}\,m\,q_u\,r}{n+m} \qquad \text{(5·4)}$$

We will now try to get the second part of equation (5·4) in terms of q and d only. For unsaturated cycles, the net rate of discharge of the queue is $s - q_u$. Thus, the vehicles disperse in time $N/(s - q_u)$ and the average queue is

$$\frac{N}{2c}\left(\frac{N}{s-q_u} + r\right).$$

The average delay d_u is therefore given by

$$d_u = \frac{r}{2c}\left(\frac{q_u r}{s - q_u} + r\right) \qquad \ldots\ldots\ldots\ldots(5{\cdot}5)$$

since N is $q_u r$ for unsaturated cycles.

Rearranging, we have

$$d_u = \frac{r}{2}\left(\frac{r\,s}{c\,(s - q_u)}\right) \simeq \frac{r}{2} \qquad \ldots\ldots\ldots\ldots(5{\cdot}6)$$

Equation (5·4) becomes

$$N = \tfrac{1}{2}\,q\,r + \frac{n\,q_s\,d_s + m\,q_u\,d_u}{n + m}$$

$$\therefore\quad N = \tfrac{1}{2}\,q\,r + q\,d \qquad \ldots\ldots\ldots\ldots(5{\cdot}7)$$

The effect of the approximation in equation (5·6) will be to give a value of N which is too small when most of the cycles are unsaturated. When most of the cycles are saturated, the effect of the approximation is small since d_u is very much smaller than d_s and m is also smaller than n. Since we know that the average queue cannot be less than $q\,r$, the general expression for N is given as :

$$N = (\tfrac{1}{2}\,q\,r + q\,d) \text{ or } q\,r\text{, whichever is the larger.} \qquad \ldots\ldots\ldots(5{\cdot}8)$$

The examples given in Table 6 justify this result.

To make this equation applicable to practical conditions a slight correction should be made. Vehicles in this investigation (see Appendix 2) were assumed to arrive at the intersection at times A, A′, etc. in Fig. 1 but in practice they would join the queue earlier than the theory assumes owing to the finite extent of the queue. Thus, the calculated values of queue will be somewhat smaller than the observed values, the difference depending on the length of the queue and the free running speed of the traffic.

Let v = free running speed of the traffic

t = time for a vehicle to travel the length of the queue at the running speed

a = number of lanes in the queue

j = average spacing of vehicles in the queue.

Now the length of the queue when the green period begins $= \dfrac{Nj}{a}$

Therefore

$$t = \frac{Nj}{av}$$

The number of vehicles which arrive in this time is

$$\frac{q\,Nj}{av}$$

Therefore, the expression for the average queue at the beginning of the green period should be

$$N = q\left(\frac{r}{2} + d\right) + \frac{q\,Nj}{av} \text{ or } qr + \frac{q\,Nj}{av}$$

$$= \frac{q\left(\frac{r}{2} + d\right)}{\left(1 - \frac{qj}{av}\right)} \text{ or } \frac{qr}{\left(1 - \frac{qj}{av}\right)}$$

Thus

$$N = q\left(\frac{r}{2} + d\right)\left(1 + \frac{qj}{av}\right) \text{ or } qr\left(1 + \frac{qj}{av}\right) \quad \text{......(5·9)}$$

APPENDIX 6

STOPS AND STARTS OF VEHICLES

Proportion of vehicles which stop at least once

Let us assume that all vehicles which arrive whilst there is still a queue of vehicles have to stop. In practice some of these vehicles would only have to slow down; the expression to be derived will therefore overestimate the effect

If T is the portion of the green period whilst the queue remains then

$$sT + q\,(g - T) = qc$$

$$T = \frac{qc - qg}{s - q}$$

$$= \frac{qr}{s - q}$$

$$= \frac{yr}{1 - y} \quad \text{................................(6·1)}$$

If R is the proportion of vehicles which stop

$$R = \frac{r + T}{c}$$

$$= \frac{r}{c\,(1 - y)} \quad \text{from equation (6·1)}$$

$$\therefore \quad R = \frac{1 - \lambda}{1 - y} \quad \text{..................................(6·2)}$$

Average number of stops and starts per vehicle

The total number of stops and starts during each cycle is equal to the number of vehicles in the queue at the beginning of the green period plus those vehicles which arrive while the queue is clearing during the green period.

If the average queue at the beginning of the green period (denoted by N) can clear during the green, i.e.

$$\text{if } \frac{N}{s - q} < g$$

then the average number of stops and starts per vehicle is

$$P = \frac{N + \dfrac{qN}{s - q}}{qc}$$

$$= \frac{Ns}{qc\,(s - q)}$$

i.e.,

$$P = \frac{N}{qc\,(1 - y)} \qquad \text{.................................(6·3)}$$

where $N = q(\frac{r}{2} + d)$ or qr, whichever is larger (equation (8)). If the average queue at the beginning of the green period cannot be fully discharged during one green time, i.e.

$$\text{if } \frac{N}{s - q} > g$$

then all vehicles which arrive during the green will be stopped as well as those which constitute the queue at the beginning of the green period. Thus,

$$P = \frac{N + qg}{qc}$$

$$P = \frac{N}{qc} + \lambda \qquad \text{......................................(6·4)}$$

where N is given by equation (8).

Using equations (6·3) and (6·4), the average number of stops and starts was calculated for several values of the degree of saturation for $\lambda = 0{\cdot}4$ and a flow of 10 vehicles per cycle. The results are shown in Fig. 9 together with a curve representing the proportion of vehicles which make at least one stop (from equation (6·2)).

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